Ambiguities in one-dimensional phase retrieval from Fourier magnitudes

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In many scientific areas, such as astronomy, electron microscopy, and crystallography, one is faced with the problem to recover an unknown signal from the magnitudes of its Fourier transform. Unfortunately, this phase retrieval problem is complicated because of the well-known ambiguousness.

Using a novel approach, we give a complete characterization of all occurring ambiguities. Moreover, we show that each further solution of the discrete-time phase retrieval problem can be described by an appropriate convolution representation of the original signal and by suitable rotations, shifts, and conjugations and reflections of the appearing factors.

Based on our characterization of the solution set, we investigate different a priori conditions in order to reduce the number of ambiguities or even to receive a unique solution. For example, if we have access to additional magnitudes or phases of the unknown signal in the time domain, we can show that almost all signals with finite support can be uniquely recovered. An analogous result can be obtained by exploiting additional interference measurements. Here we study the interference of the unknown signal with an unknown reference signal.