Computational Tools for PDEs with Random Coefficients

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The simulation and forecast of complex physical processes in science, engineering and industry requires input data which are often subject to considerable uncertainties. This is due to incomplete models, measurement errors or lack of knowledge. Partial differential equations (PDEs) with random coefficients offer the opportunity to incorporate data uncertainties in mathematical models and subsequent computer simulations. However, these PDEs are formulated in a physical domain coupled with a possibly high-dimensional sample space generated by random parameters and can be very computing-intensive.

We discuss a model elliptic PDE of single phase subsurface flow in a random porous medium. The diffusion coefficient is modeled as a lognormal random field with rough realisations. Hence a large number of random parameters is required. To date, only Monte Carlo based methods are computationally feasible in this case, however the cost of Monte Carlo is often prohibitively large. We employ multilevel Monte Carlo (MLMC), a novel variance reduction technique which can reduce the cost significantly. We explain the basic MLMC idea and combine this technique with mixed finite element discretisations to calculate travel times of particles in groundwater flows. As a second example, we will outline a novel multilevel estimator for rare events based on subset simulation.