A PDE approach to fractional diffusion

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The purpose of this talk is the study of solution techniques for problems involving fractional powers of symmetric, coercive and elliptic operators. These operators can be realized as the Dirichlet to Neumann map of a nonuniformly elliptic problem posed on a semi-infinite cylinder, which we analyze in the framework of weighted Sobolev spaces. Motivated by the rapid decay of the solution of this problem, we propose a truncation that is suitable for numerical approximation. We discretize this truncation using first degree tensor product finite elements. We derive suboptimal a priori error estimates for quasi-uniform discretizations and quasi-optimal error estimates for anisotropic discretizations, both estimates in weighted Sobolev spaces. Next, we explore extensions and applications of the a priori theory previously described: a posteriori error analysis and adaptivity, the elliptic fractional obstacle problem, parabolic equations with fractional diffusion and Caputo fractional time derivative and optimal control problems.