Hyperbolic Systems with Dynamic Boundary Conditions*

Gilbert R. Peralta
Institute for Mathematics and Scientific Computing
University of Graz

Abstract

We consider first order hyperbolic systems on an interval with dynamic boundary conditions. These systems occur when the dynamics on the boundary interact with the waves in the interior. The well-posedness for linear systems is established using the method of Friedrichs. Due to the limited regularity of the coefficients we need to introduce the appropriate test functions for the weak formulation. The weak solutions satisfy a hidden regularity at the boundary and this cannot be achieved from standard semigroup methods. As a consequence, the state component at the boundary satisfies an additional regularity. The linear theory is used to analyze the local-in-time well-posedness for nonlinear systems. As an application, we consider a model describing the flow of a fluid inside an elastic tube whose ends are attached to tanks. We prove global existence when the data is smooth enough and close to the steady state. In particular, the rate of convergence to the equilibrium for the nonlinear system is the same as that of the linear one. Entropy and energy methods will be utilized to prove this result. The main difficulty here is the boundedness of the domain and the coupling to ODE boundary conditions.

*Joint work with Georg Propst, Institute for Mathematics and Scientific Computing, University of Graz