A quantitative theory in stochastic homogenization

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In many applications, one has to solve an elliptic equation with coefficients that vary on a length scale much smaller than the domain size. We are interested in a situation where the coefficients are characterized in statistical terms: Their statistics are assumed to be translation invariant and to decorrelate over large distances. As is known by qualitative theory, the solution operator behaves -- on large scales -- like the solution operator of an elliptic problem with homogeneous, deterministic coefficients!

We are interested in several quantitative aspects: How close is the actual solution to the homogenized one -- we give an optimal answer in terms of the quenched Green's function, and point out the connections with elliptic regularity theory (input from Nash's theory, a new outlook on De Giorgi's theory).

We are also interested in the quantitative ergodicity properties for the process usually called ``the environment as seen from the random walker''. We give an optimal estimate that relies on a link with (the Spectral Gap for) another stochastic process on the coefficient fields, namely heat-bath Glauber dynamics. This connection between statistical mechanics and stochastic homogenization has previously been used in opposite direction (i.e. with qualitative stochastic homogenization as an input).

Theory provides a formula for the homogenized coefficients, based on the construction of a “corrector", which defines harmonic coordinates. This formula has to be approximated in practise, leading to a random and a systematic error. If time permits, we point out optimal estimates of both.