

Karl-Franzens-Universität Graz
A Priori Error Analysis for Discretization of Sparse Elliptic
Optimal Control Problems in Measure space

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Abstract

We consider an elliptic optimal control problem with a sparsity functional, where the optimal control is searched for in the space of regular Borel measures. It has been observed, see e.g. [1], that the solutions of these problems have highly sparse solutions, which has promising applications for the optimal placement of actuators and in the context of inverse problems.

For the numerical realization, the problem is discretized with finite elements, where we follow the recent work [2]. However, the a priori convergence rates obtained so far are not optimal. In [3] we significantly improve these rates in the case of two spacial dimensions and also provide analysis for the three dimensional case. The theoretical convergence results we obtain agree with the generic regularity of the solutions as well as with the numerical observations.

Furthermore, in specific cases, additional regularity for the optimal controls can be obtained by careful inspection of the optimality system. In practice this results in an improved order of convergence for the problem in three dimensions. Motivated by this, we also obtain an improved estimate under these conditions.

References

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